Schematics of Brownian Motion and Fluctuation - Dissipation Theory

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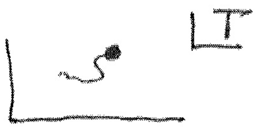
- mixing decay relation
- entropy balance relation

→ Now, have discussed structure of diffusion and its relation to underlying pdf. However have not addressed physics → i.e. relation to dynamics.
i.e. what physics sets D, Δx, Δt etc.
This brings us to:

Brownian Motion

- classic example of diffusion arises in random walk of particle, driven by thermal, random kicks, and restricted by drag



small particle
$l \Rightarrow$ scale
$\eta = \nu\rho \Rightarrow$ viscosity

$$m\frac{dv}{dt} = -\beta v + \tilde{f}$$

m: particle mass

$\beta \sim (6\pi\eta l)$

$\tilde{f}$: Brownian force random.
additive noise

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Random force → no memory

Correlation fctn.

$$\langle \tilde{f}(t_1)\,\tilde{f}(t_2)\rangle = \underset{\text{strength}}{\tilde{f}^2}\,\tau_{ac}\,\delta(t_2-t_1)$$

τ_{ac} → required for dimension

→ memory time of force, necessarily shortest in problem

⟺ delta-correlated force.

$$\langle \tilde{f}^2\rangle_\omega = \int e^{-i\omega(t_2-t_1)}\langle \tilde{f}(1)\tilde{f}(2)\rangle\, d(t_2-t_1)$$

dims $1/\omega$

$$= \tilde{f}^2\tau_{ac} \;\Rightarrow\; \sim \text{const} \;\rightarrow\; \text{"white noise"}$$

Now, what is $\tilde{f}$ ⟺ relate to temperature

↳ force strength.

$$m_p \frac{dv}{dt} = -\beta v + \tilde{f}$$

n.b.

strength $\tilde{f}^2$

$$\int \frac{d\tau}{\tau_{ac}}\langle \tilde{f}(0)\tilde{f}(\tau)\rangle = \langle \tilde{f}^2\rangle$$

steady state ⟹

$$\underset{\substack{\text{power dissipated}\\ \text{by drag}}}{\beta\langle \tilde{v}^2\rangle} \sim \underset{\substack{\text{power input by}\\ \text{Brownian force.}}}{\langle \tilde{f}\,\tilde{v}\rangle}$$

at $\boxed{T}$

Argument ⟹ Fluctuation/Dissp'n Thm.

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8) $\beta\langle\tilde{v}^2\rangle \sim \langle\tilde{f}\tilde{v}\rangle$

but $m_p\langle\tilde{v}^2\rangle \sim T$ → bath sets thermal reservoir!!

$$\langle\tilde{v}^2\rangle \sim \frac{T}{m_p} \sim \frac{\langle\tilde{f}\tilde{v}\rangle}{\beta} \sim \frac{\langle\tilde{f}^2\rangle}{\beta^2}$$

$\Rightarrow$ $\boxed{\langle\tilde{f}^2\rangle \sim \beta^2 T/m_p.}$ → sets Brownian force.

For diffusion of Brownian particle.

$$D \sim (\Delta x)^2/\tau \sim \langle\tilde{v}^2\rangle\,\tau$$

↓
spatial diffusion.

Velocity relaxes as β/m_p rate.

$\Rightarrow$

$$D \sim \langle\tilde{v}^2\rangle\frac{m_p}{\beta} \sim \frac{T}{m_p}\frac{m_p}{\beta} \sim T/\beta$$

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Fluctuation - Dissipation Thm.

→ Drag-induced energy dissipation balances fluctuation work at steady state, to maintain temp. T.

→ given any 2 of T, drag, fluctuations (force), can deduce third.

i.e. $D = T/\beta \iff D = \mu T$

but $D \sim \langle \delta \dot{v}^2 \rangle / \tau_c$

$\sim \langle \delta V^2 \rangle \tau_c$

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$\Rightarrow \boxed{D \sim T/6\pi\mu\ell}$ → diffusivity in space of Brownian particle.

$D \sim T/\beta$

recall: $F_d \sim \beta v$.

$D = \mu T \qquad \mu \sim 1/(n v_{Th} \sigma_L) M_L$

Similar, not identical.

what of pdfs?

→ velocity, i.e. $P(v, t)$ for Brownian particle?

recall,
$$M_p \frac{dv}{dt} = -\beta v + \tilde{f}$$

or equivalently:

$$\frac{dv}{dt} = -\alpha v + \tilde{a} \qquad \alpha = \beta/M_p$$

$\tilde{a} \equiv$ accel.